Assignment No. - 1

• Matrix - 1 •

Q 1 Find the rank of following matrices by normal form.

1] $A = \begin{bmatrix} 5 & -2 & 4 & -1 & 6 \\ -2 & 1 & 1 & -2 & -2 \\ 4 & 1 & 0 & 5 & 10 \\ -1 & -2 & 5 & -8 & -6 \end{bmatrix}$

$(R_1 - R_3)$; $(R_3 + 2R_2)$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & -4 \\ -2 & 1 & 1 & -2 & -2 \\ 0 & 3 & 2 & 1 & 6 \\ -1 & -2 & 5 & -8 & -6 \end{bmatrix}$

$C_5 \to -\frac{1}{2} C_5$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & 2 \\ -2 & 1 & 1 & -2 & 1 \\ 0 & 3 & 2 & 1 & -3 \\ -1 & -2 & 5 & -8 & 3 \end{bmatrix}$

$(R_2 - 2R_4)$; $(R_4 + R_1)$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & 2 \\ 0 & 5 & -9 & 14 & -5 \\ 0 & 3 & 2 & 1 & -3 \\ 0 & -5 & 9 & -14 & 5 \end{bmatrix}$

$R_4 + R_2$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & 2 \\ 0 & 5 & -9 & 14 & -5 \\ 0 & 3 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$(C_5 + C_2)$;

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & 2 \\ 0 & 5 & -9 & 14 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$(C_2 + 3C_1)$; $(C_3 - 4C_1)$; $(C_4 + 6C_1)$; $(C_5 - 2C_1)$

$A = \begin{bmatrix} 1 & 0 & 0 & -0 & 0 \\ 0 & 5 & -9 & 14 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2 \to \frac{1}{5} R_2$

$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -9/5 & 14/5 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_3 - 3R_2$

$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -9/5 & 14/5 & 0 \\ 0 & 0 & -37/5 & 22/5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$(C_3 + 9/5\, C_2)$; $(C_4 - 14/5\, C_2)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -37/5 & 22/5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$(C_3 \to -\frac{5}{37} C_3)$; $(C_4 \to \frac{5}{22} C_4)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 - C_4$

$$A = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\approx \left[\begin{array}{c|c} I_3 & 0 \\ \hline 0 & 0 \end{array}\right]$$

The above matrix is in normal form with rank 3.

2] $$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{bmatrix}$$

$(R_2 - R_1)$; $(R_3 - R_1)$; $(R_4 - R_1)$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 2 & -2 & 0 \\ 0 & -3 & -1 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$(C_2 - C_1)$; $(C_3 - 2C_1)$; $(C_4 - 3C_1)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -3 & -1 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -3 & -6 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to -C_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -6 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 + 2R_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 4 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(C_3+3C_2) ; (C_3+6C_2)

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&4&-12\\0&0&0&0\end{bmatrix}$$

$(C_3 \to \frac{1}{4}C_3)$; (C_4+12C_3)

$$A=\left[\begin{array}{ccc:c}1&0&0&0\\0&1&0&0\\0&0&1&0\\ \hdashline 0&0&0&0\end{array}\right]$$

$$\approx\left[\begin{array}{c:c}I_3&0\\0&0\end{array}\right]$$

The above matrix is in normal form with rank 3

3) $$A=\begin{bmatrix}1&2&-1&2\\2&2&-1&1\\-1&-1&1&-1\\2&1&-1&2\end{bmatrix}$$

(R_2-2R_1) ; (R_3+R_1) ; (R_4-2R_1)

$$A=\begin{bmatrix}1&2&-1&2\\0&-2&1&-3\\0&1&0&1\\0&-3&1&-2\end{bmatrix}$$

(C_2-2C_1) ; (C_3+C_1) ; (C_4-2C_1)

$$A=\begin{bmatrix}1&0&0&0\\0&-2&1&-3\\0&1&0&1\\0&-3&1&-2\end{bmatrix}$$

$R_3 \leftrightarrow R_2$

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&-2&1&-3\\0&-3&1&-2\end{bmatrix}$$

(R_3+2R_2) ; (R_4+3R_2)

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&1\\0&0&1&1\\0&0&1&1\end{bmatrix}$$

C_4-C_2

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&1\\0&0&1&1\end{bmatrix}$$

(C_4-C_3)

$$A=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&1\\0&0&0&0\end{bmatrix}$$

$C_4 - C_3$

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$$\approx \left[\begin{array}{c|c} I_3 & 0 \\ \hline 0 & 0 \end{array}\right]$$

The above matrix is in normal form with rank 3

4) $A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$

$(R_2 - R_1)$; $(R_3 - R_2)$; $(R_4 - R_3)$

$$A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$$

$(R_2 - 3R_1)$; $(R_3 - R_1)$; $(R_4 - R_1)$
$(R_4 - R_1)$; $(R_5 - 15R_1)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$(C_2 - C_1)$; $(C_3 - C_1)$; $(C_4 - C_1)$; $(C_5 - C_1)$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

$R_5 - R_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$(C_3 - 2C_2)$; $(C_4 - 3C_2)$; $(C_5 - 4C_2)$

$$A = \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$A \approx \left[\begin{array}{c|c} I_2 & 0 \\ 0 & 0 \end{array}\right]$$

The above matrix is in normal form with rank 2.

Q2 Find the rank of following matrices by Echelon form.

1] $A = \begin{bmatrix} 5 & -2 & 4 & -1 & 6 \\ -2 & 1 & 1 & -2 & -2 \\ 4 & 1 & 0 & 5 & 10 \\ -1 & -2 & 5 & -8 & -6 \end{bmatrix}$

$R_1 - R_3$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & -4 \\ -2 & 1 & 1 & -2 & -2 \\ 4 & 1 & 0 & 5 & 10 \\ -1 & -2 & 5 & -8 & -6 \end{bmatrix}$

$(R_2 + 2R_1)$; $(R_3 - 4R_1)$; $(R_4 + R_1)$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & -4 \\ 0 & -5 & 9 & -14 & -10 \\ 0 & 13 & -16 & 35 & 26 \\ 0 & -5 & 9 & -14 & -10 \end{bmatrix}$

$(R_2 - R_4)$; $\cancel{(R_4 - R_2)}$

$A = \begin{bmatrix} 1 & -3 & 4 & -6 & -4 \\ 0 & -5 & 9 & -14 & -10 \\ 0 & 13 & -16 & 35 & 26 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\cancel{R_2 \leftrightarrow R_3}$

$A = \left[\begin{array}{cc|ccc} 1 & -3 & 4 & -6 & -4 \\ 0 & -5 & 9 & -14 & -10 \\ 0 & 13 & -16 & 35 & 26 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$$A \approx \left[\begin{array}{c|c} I_3 & H \\ 0 & 0 \end{array}\right]$$

The above matrix is in Echelon form with rank 3

2] $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{bmatrix}$

$(R_2 - R_1)$; $(R_3 - R_1)$; $(R_4 - R_1)$

$A = \left[\begin{array}{cc|cc} 1 & 1 & 2 & 3 \\ 0 & 2 & -2 & 0 \\ 0 & -3 & -5 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$$\approx \left[\begin{array}{c|c} G & H \\ 0 & 0 \end{array}\right]$$

The above matrix is in Echelon form with rank 3

3) $$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 1 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

$(R_2 - 2R_1)$; $(R_3 + R_1)$; $(R_4 - 2R_1)$

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{bmatrix}$$

$(R_2 \leftrightarrow R_3)$

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & -3 & 1 & -2 \end{bmatrix}$$

$(R_3 + 2R_2)$; $(R_4 + 3R_2)$

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(R_4 - R_3)$

$$A = \left[\begin{array}{cc|cc} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\approx \left[\begin{array}{c|c} G & H \\ 0 & 0 \end{array}\right]$$

The above matrix is in Echelon form with rank 3

4) $$A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$$

$(R_2 - R_1)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 8 & 9 & 10 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$$

$(4R_1 - R_2)$; $(5R_1 - R_3)$; $(6R_1 - R_4)$; $(15R_1 - R_5)$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 & -4 \\ 0 & -2 & -3 & -4 & -5 \\ 0 & -3 & -4 & -5 & -6 \\ 0 & -1 & -2 & -3 & -4 \end{bmatrix}$$

(R_3+2R_2); (R_3+3R_2); (R_4+R_2)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(R_4-2R_3) ; (R_2-R_3)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(R_1-R_3)

$$A = \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$A \approx \left[\begin{array}{c|c} G & H \\ 0 & 0 \end{array}\right]$$

Above matrix is in Echelon form with rank 2.

Q3 Test for consistancy, and if possible, solve the following equation.

1] $2x+3y+4z=11$
$x+5y+7z=15$
$3x+11y+13z=25$

The matrix eq^n is $AX=B$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 7 \\ 3 & 11 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 25 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 5 & 7 \\ 2 & 3 & 4 \\ 3 & 11 & 13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 11 \\ 25 \end{bmatrix}$$

(R_2-2R_1); (R_3-3R_1)

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -10 \\ 0 & -4 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 19 \\ -20 \end{bmatrix}$$

$(7R_3-4R_2)$

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & -7 & -10 \\ 0 & 0 & -16 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ -19 \\ -64 \end{bmatrix}$$

Here, we find that,

Rank of A = Rank of [A:B]

Hence eq^{ns} are consistent

Also rank of A = No. of variables

$\therefore$ Matrix will have an unique solution.

$\therefore\ x + 5y + 7z = 15$

$-7y - 10z = -19$

$-16z = -64$

Now,

$16z = 64$

$\therefore \boxed{z = 4}$

Also, $-7y - 40 = -19$

$-7y = 21$

$\boxed{y = -3}$

And, $x - 15 + 28 = 15$

$x = 15 - 13$

$\boxed{x = 2}$

$\therefore$ Hence unique sol^n is

$x = 2$; $y = -3$; $z = 4$

2) $3x - y + 4z = 3$

$x + 2y - 3z = -2$

$6x + 5y + z = -3$

The matrix eq^n is $AX = B$

$$\begin{bmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 4 \\ 6 & 5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -3 \end{bmatrix}$$

$(R_2 - 3R_1)$; $(R_3 - 6R_1)$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -7 & 13 \\ 0 & -7 & 19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ 9 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -7 & 13 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix} \quad \ldots (I$$

Here, We find that,

Rank of A = Rank of [A:B]

Hence eq^{ns} are consistent

Also, rank of A = No. of variab

Hence, matrix eq^n (I) will have unique sol^n

$\therefore\ x + 2y - 3z = -2$

$-7y + 13z = 9$

$6z = 0$

$\therefore \boxed{z = 0}$

$\therefore -7y + 13z = 9$

$\therefore \boxed{y = -9/7}$

$x + 2y - 3z = -2$

$x + 2(-9/7) - 3(0) = -2$

$\therefore \boxed{x = 4/7}$

Hence unique sol^n is
$x = 4/7$; $y = -9/7$; $z = 0$

3) $2x - 3y + 7z = 5$
$3x + y - 3z = 13$
$2x + 19y - 47z = 32$

The matrix eq^n is $AX = B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$R_1 \to (R_2 - R_1)$

$$\begin{bmatrix} 1 & 4 & -10 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 13 \\ 32 \end{bmatrix}$$

$(R_2 - 3R_1)$; $(R_3 - 2R_1)$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 11 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 16 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 4 & -10 \\ 0 & -11 & 27 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -11 \\ 5 \end{bmatrix}$$

Here, we find that,
Rank of A ≠ Rank of [A:B]
Hence eq^{ns} are incosistant.
∴ They will have no sol^n.

4) $x + y + z = 3$
$2x - y + 3z = 1$
$4x + y + 5z = 2$
$3x - 2y + z = 4$

The matrix eq^n is $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 1 & 5 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 4 \end{bmatrix}$$

$(R_2 - 2R_1)$; $(R_3 - 4R_1)$; $(R_4 - 3R_1)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -3 & 1 \\ 0 & -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ -10 \\ -5 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & 0 \\ 0 & -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ -5 \\ -5 \end{bmatrix}$$

$R_4 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & -5 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ -5 \\ -5 \end{bmatrix}$$

Here, We find that,
Rank of A $\neq$ Rank of B
Hence eq^{ns} are inconsistant
$\therefore$ They will have no sol^n

Q4 Solve the following and check for consistency.

1) $2x - y - z = 2$

$x + 2y + z = 2$

$4x - 7y - 5z = 2$

Matrix eq^n is $AX = B$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$R_2 \longleftrightarrow R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & -1 \\ 4 & -7 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$(R_2 - 2R_1)$; $(R_3 - 4R_1)$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -3 \\ 0 & -15 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -6 \end{bmatrix}$$

$R_3 - 3R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -5 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

Here, We find that
Rank of matrix A = Rank of [A : B]
But,
Rank of A = 2 < 3 = no. of variabl
$\therefore$ No. of parameters = 3 - 2 = 1
Also eq^n has infinite sol^{ns}.

$x + 2y + z = 2$

$-5y - 3z = -2$

Let $z = t$

$\therefore y = \dfrac{-2+3t}{-5}$

$y = \dfrac{2-3t}{5}$

Now,

$x = 2 - 2y + z$

$= 10 - 4 + 6t + 5t$

$\therefore x = \dfrac{6-11t}{5}$

$\therefore$ Infinite sol^{ns} are

$x = \dfrac{6-11t}{5}$; $y = \dfrac{2-3t}{5}$; $z = t$

2) $x - 2y + 2z = 7$

$2x + z = 4$

$3x + 2y = 1$

Matrix eq^n is $AX = B$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ 1 \end{bmatrix}$$

$(R_2 - 2R_1)$; $(R_3 - 3R_1)$

$$\begin{bmatrix} 1 & -2 & 2 \\ 0 & 4 & -3 \\ 0 & 8 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \\ -20 \end{bmatrix}$$

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & -2 & 2 \\ 0 & 4 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \\ 0 \end{bmatrix} \quad \ldots (I)$$

Here, Rank of A = Rank of $[A:B]$

But rank of $A = 2 < 3$ = no. of variable

$\therefore$ No. of parameters $= 3 - 2 = 1$

$x - 2y + 2z = 7$

$4y - 3z = -10$

Let $z = t$

$\therefore y = \dfrac{-10+3t}{4}$

$x = 7 + 2y - 2z$

$x = \dfrac{4-t}{2}$

$\therefore$ Infinite sol^{ns} are

$x = \dfrac{4-t}{2}$; $y = \dfrac{3t-10}{4}$; $z = t$

3] $5x_1 + 3x_2 + 7x_3 = 4$

$3x_1 + 26x_2 + 2x_3 = 9$

$7x_1 + 2x_2 + 10x_3 = 5$

The matrix eq^n is $AX = B$

$$\begin{bmatrix} 6 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

$3R_1 - 2R_3$

$$\begin{bmatrix} 1 & 5 & 1 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \\ 6 \end{bmatrix}$$

$(R_2 - 3R_1)$; $(R_3 - 7R_1)$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & 11 & -1 \\ 0 & -33 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -9 \end{bmatrix}$$

$R_3 + 3R_2$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & 11 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

Here, we find that,
Rank of A = Rank of $[A \vdots B]$
$\therefore$ The eq^{ns} are consistent.
but,
Rank of A = $3 < 2$ = no. of variable
Hence, it has infinite sol^n
$\therefore$ No. of parameters = 1

$\therefore x_1 + 5x_2 + x_3 = 2$
$11x_2 - x_3 = 3$

Let $x_3 = t$
then,
$11x_2 - x_3 = 3$
$11x_2 - t = 3$
$11x_2 = 3 + t$
$x_2 = \frac{3+t}{11}$

Now,
$x_1 + 5x_2 + x_3 = 2$

$x_1 = \frac{22 - 15 - 5t - 11t}{11}$

$x_1 = \frac{7 - 16t}{11}$

$\therefore$ An infinite sol^n is

$x_1 = \frac{7-16t}{11}$; $x_2 = \frac{3+t}{11}$; $x_3 = t$

Q5 Find the value of λ if following values are consistant also solve eq^n for obtained value of λ.

$$x_1 + x_2 + x_3 = 1$$
$$2x_1 + x_2 + 4x_3 = \lambda$$
$$4x_1 + x_2 + 10x_3 = \lambda^2$$

$\rightarrow$ $$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$(R_2 - 2R_1)$; $(R_3 - 4R_1)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & -3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda-2 \\ \lambda^2-4 \end{bmatrix}$$

$(R_3 - 3R_2)$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda-2 \\ \lambda^2-3\lambda+2 \end{bmatrix} \quad \text{....(I)}$$

It is given that, eq^n are consistance i.e.
Rank of $[A]$ = Rank of $[A \vdots B]$
This is possible only if we put $(\lambda^2 - 3\lambda + 2) = 0$

$\lambda(\lambda - 3) = -2$

$\lambda = 2, 1$

Case-(I) $\lambda = 2$

$\therefore$ (I) $\Rightarrow$ $$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

We find that,
Rank of matrix $A = 2 < 3$ = no. of Variables

$\therefore$ Eq^{ns} will have infine sol^{ns}
No. of parameters = 1

$\therefore$ $x_1 + x_2 + x_3 = 1$
$-x_2 + 2x_3 = 0$

let $x_3 = t$
$x_2 = 2t$

Also,
$x_1 = -2t - t$
$x_1 = -3t$

$\therefore$ Infinite sol^{ns} are
$x_1 = -3t$, $x_2 = 2t$, $x_3 = t$

Case-(II): For $\lambda = 1$

$$\therefore (I) \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

We find that,

Rank of $[A] = 2 < 3 =$ no. of variables

$\therefore$ Eq^n will have infine sol^n

$\therefore$ No. of parameters = 1

$$\therefore \quad x_1 + x_2 + x_3 = 1$$
$$-x_2 + 2x_3 = -1$$

$\therefore$ let $x_3 = t$

$$\therefore \quad -x_3 + 2t = -1$$
$$-x_2 = -1 - 2t$$
$$x_2 = 1 + 2t$$

Also,

$$x_1 + x_2 + x_3 = 1$$
$$\therefore x_1 = 1 - x_2 - x_3$$
$$\therefore x_1 = t$$

$\therefore$ Infinite sol^{ns} are

$x_1 = t$, $x_2 = 1 + 2t$, $x_3 = t$

Q6 Find values of α & β if $3x - 2y + z = \beta$, $5x - 8y + 9z = 3$, $2x + y + \alpha z = -1$ has

(i) Unique sol^n (ii) No sol^n (iii) infinite sol^{ns}

$$\rightarrow \begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \beta \\ 3 \\ -1 \end{bmatrix}$$

$R_1 - R_3$

$$\begin{bmatrix} 1 & -3 & 1-\alpha \\ 5 & -8 & 9 \\ 2 & 1 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \beta+1 \\ 3 \\ -1 \end{bmatrix}$$

$(R_2 - 5R_1)\ ;\ (R_3 - 2R_1)$

$$\begin{bmatrix} 1 & -3 & 1-\alpha \\ 0 & 7 & 4-5\alpha \\ 0 & 7 & -2+3\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \beta+1 \\ -\beta-2 \\ -2\beta-3 \end{bmatrix}$$

$(R_3 - R_2)$

$$\begin{bmatrix} 1 & -3 & 1-\alpha \\ 0 & 7 & 4-5\alpha \\ 0 & 0 & 8\alpha-6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \beta+1 \\ -\beta-2 \\ -\beta-1 \end{bmatrix}$$

(i) Unique Sol^n -

Matrix eq^n (I) will be consistance and also it will have unique sol^n if & only if

$8\alpha - 6 \neq 0$ & $-\beta - 1 = 0$

$\boxed{\alpha \neq 3/4}$ & $\boxed{\beta = -1}$

(ii) No sol^n -

The matrix eq^n (I) will have no sol^n if and only if rank of $[A] \neq$ Rank of $[A \vdots B]$

This is possible if

$8\alpha - 6 = 0$ & $-\beta - 1 \neq 0$

$\boxed{\alpha = 3/4}$ & $\boxed{\beta \neq -1}$

(iii) Infinite sol^n -

The matrix eq^n (I) will have infinite sol^n if and only if $[A] <$ no. of variable also rank of A and rank of B should be equal.

$\therefore$ If we put,

$8\alpha - 6 = 0$ & $-\beta - 1 = 0$

$\therefore \boxed{\alpha = 3/4}$ & $\boxed{\beta = -1}$

Q7 Solve the following eq^{ns} by matrix method.

1) $4x_1 - x_2 + x_3 = 0$

$x_1 - 2x_2 - x_3 = 0$

$3x_1 + x_2 + 5x_3 = 0$

$\therefore$ Matrix eq^n is $AX = B$

$$\therefore \begin{bmatrix} 4 & -1 & 1 \\ 1 & -2 & -1 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \longleftrightarrow R_1$

$$\begin{bmatrix} 1 & -2 & -1 \\ 4 & -1 & 1 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(R_2 - 4R_1)$; $(R_3 - 3R_1)$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & -9 & 5 \\ 0 & -5 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(9R_3 - 5R_2)$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -9 & 5 \\ 0 & 0 & 47 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank of A = No. of Variables

$\therefore$ Matrix will have zero sol^n

$\therefore x_1 = 0$; $x_2 = 0$; $x_3 = 0$

2) $4x + 3y - z = 0$

$3x + 4y + z = 0$

$x - y - 2z = 0$

$\therefore$ Matrix eq^n is $AB = 0$

$$\begin{bmatrix} 4 & 3 & -1 \\ 3 & 4 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(R_1 - R_2)$

$$\begin{bmatrix} 1 & -1 & -2 \\ 3 & 4 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$(R_2 - 3R_1)$; $(R_3 - R_1)$

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 7 & 7 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

As the rank of $A = 2 < 3 =$ no. of variables

It will have non-zero sol^n, it will have infinite sol^{ns}

$\therefore$ No. of parameters = 1

$\therefore x - y - 2z = 0$

$7y + 7z = 0$

i.e $z = t$

$y + t = 0$

$\therefore y = -t$

Now, $x = y + 2z$

$= -t + 2t$

$= t$

$\therefore$ Infinite sol^n are

$x = t, \; y = -t, \; z = t$

Last Q Remaining!