Tutorial No.-1

• Rivision and collection of basic Formulae in Mathematics.

1] Trigonometric Formulae

1] $\sin^2\theta + \cos^2\theta = 1$

2) $\sec^2\theta = 1 + \tan^2\theta$

3) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

4) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

5) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

6) $\sin 2\theta = 2\sin\theta\cos\theta$

7) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

8) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

9) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

10) $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

11) $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

12) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

13) $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cdot\cos\left(\dfrac{A-B}{2}\right)$

14) $\sin A - \sin B = 2\cos\left(\dfrac{A+B}{2}\right)\sin\left(\dfrac{A-B}{2}\right)$

15) $\cos A + \cos B = 2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$

16) $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

17) $2\sin A.\cos B = \sin(A+B) + \sin(A-B)$

18) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

19) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

20) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

21) $\cosh^2 x - \sinh^2 x = 1$

22) $\operatorname{sech}^2 x = 1 - \tanh^2 x$

23) $\operatorname{cosech}^2 x = \coth^2 x - 1$

24) $\sinh 2x = 2\sinh x - \cosh x$

25) $\cosh 2x = 2\cosh^2 x - 1$
$= 2\sinh^2 x + 1$

2] Laws of Indices

1) $a^m . a^n = a^{m+n}$

2) $a^{m-n} = \frac{a^m}{a^n}$

3) $a^m b^m = (ab)^m$

4) $a^0 = 1$

5) $a^{-m} = \frac{1}{a^m}$

3] Laws of Logarithm

1) $\log_e(mn) = \log_e(m) + \log_e(n)$

2) $\log_e(m/n) = \log_e(m) - \log_e(n)$

3) $\log_e(m^n) = n \log_e(m)$

4) Change of base property:-

$$\log_b(m) = \frac{\log_a m}{\log_a b}$$

4] Quadratic Equations

1) The general form of quadratic equation is $ax^2+bx+c=0$ where, $a \neq 0$ and a, b, c are constant then general Sol^n of this eq^n is

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

2) If α & β are the roots of quadratic eq^n then $x^2+bx+c=0$ then formation of eq^n is given by

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

where, sum of roots $= \alpha + \beta = -b/a$

product of roots $= \alpha.\beta = c/a$

3) Nature of roots :-

If $\Delta = b^2-4ac = 0$, then roots are real & repeated

If $\Delta = b^2-4ac > 0$, then roots are real & distinct

If $\Delta = b^2-4ac < 0$, then roots are complex

5] Binomial Theorem

1) $(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_n a^0 b^n$

where, ${}^nC_r = \frac{n!}{(n-r)!\,r!}$, $n \in N$

2) $(a-b)^n = {}^nC_0 a^n b^0 - {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + (-1)^n {}^nC_n a^0 b^n$

3) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots$

4) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$

5) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

6] Algebric Formulae

1) $(a^2 - b^2) = (a-b)(a+b)$
2) $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$
3) $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$
4) $(a^4 - b^4) = (a^2+b^2)(a-b)(a+b)$
5) $(a+b)^2 = a^2 + 2ab + b^2$
6) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
7) $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$
8) $(a-b)^2 = a^2 - 2ab + b^2$
9) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
10) $(a+b+c)^2 = a^2 + 2a(b+c) + (b+c)^2$

10) $(a+b+c)^2 = a^2 + 2ab + 2ac + b^2 + c^2 + 2bc$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

11) $(a+b+c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3ac^2 + 3a^2c + 3b^2c + 3bc^2 + 6abc$

12) $a^2 + b^2 = (a+b)^2 - 2ab$

$= (a-b)^2 + 2ab$

7] Partial fraction method

- Rules for partial fraction

Case (I): If D contains linear non repeated factors i.e. If D contains $(x+a)(x+b)$ then its partial fraction is

$$P.F. = \frac{A}{x+a} + \frac{B}{x+b}$$

Case (II): If D contains linear repeated factors i.e $(x+a)$ is repeated three times then its partial fraction is

$$P.F. = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3}$$

Case(III): If D contains quadratic factors then its partial fraction is

$$P.F. = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+E}{qx^2+px+c}$$

Case (IV) : If D contains repeated quadratic factor i.e. ax^2+bx+c is repeated two times then its partial fraction is

$$P.F. = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+E}{(ax^2+bx+c)^2}$$

8] Differentiation formulae

1) $\frac{d}{dx}(x^n) = nx^{n-1}$

2) $\frac{d}{dx}(c) = 0$

3) $\frac{d}{dx} e^x = e^x$

4) $\frac{d}{dx} \sin x = \cos x$

5) $\frac{d}{dx} \cos x = -\sin x$

6) $\frac{d}{dx} \tan x = \sec^2 x$

7) $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x$

8) $\frac{d}{dx} \sec x = \sec x \tan x$

9) $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

10) $\frac{d}{dx} \log_e x = \frac{1}{x}$

11) $\frac{d}{dx} a^x = a^x \log e$

12) $\frac{d}{dx} \log_a(x) = \frac{1}{x \log a}$

13) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

14) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

15) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$

16) $\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$

17) $\frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$

18) $\frac{d}{dx} \operatorname{cosec}^{-1} x = \frac{-1}{x\sqrt{x^2-1}}$

9) Integration Formulae

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

2) $\int \sin x \, dx = -\cos x + C$

3) $\int \cos x \, dx = \sin x + C$

4) $\int \tan x \, dx = \log \sec x + C$

5) $\int \operatorname{cosec} x \, dx = \log [\operatorname{cosec} x - \cot x] + C$

6) $\int \sec x \, dx = \log (\sec x + \tan x) + C$

7) $\int \cot x \, dx = \log \sin x + C$

$= -\log \cos x + C$

8) $\int e^x dx = e^x + C$

9) $\int \frac{1}{x} dx = \log x + C$

10) $\int a^x dx = \frac{a^x}{\log a} + C$

11) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

12) $\int \frac{1}{\sqrt{1-x^2}} dx = -\cos^{-1} x + C$

13) $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

14) $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$

15) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$

16) $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

17) $\int \sec^2 x\,dx = \tan x + c$

18) $\int \sec x \tan x\,dx = \sec x + c$

19) $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

20) $\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \log\left(x + \sqrt{x^2 - a^2}\right) + c$

21) $\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \log\left(x + \sqrt{x^2 + a^2}\right) + c$

22) $\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

23) $\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left[\frac{a + x}{a - x}\right] + c$

24) $\int \frac{dx}{x\sqrt{x^2 - 1}} = \sec^{-1} x + c$

25) $\int \sinh x\,dx = \cosh x + c$

26) $\int \cosh x\,dx = \sinh x + c$

27) $\int \tanh x\,dx = \log \cosh x + c$

28) $\int \operatorname{sech} x\,dx = \sin^{-1}(\tanh x) + c$

29) $\int \coth x\,dx = \log \sinh x + c$

30) $\int e^x (f(x) + f'(x))\,dx = e^x f(x) + c$

31) $\int e^{ax} \sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a \sinh x - b \cos bx) + c$

32) $\int e^{ax} \cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + c$

33) $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

34) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+c$

35) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)+c$

36) $\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$

37) $\int e^{f(x)} f'(x)\,dx = e^{f(x)} + c$

38) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

39) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

40) $\int I.II\,dx = I\int II\,dx - \int\left[\frac{dI}{dx}\int II\,dx\right]dx$

- Generalised by parts rule

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$$

10] Mensuration

1) Perimeter of triangle = $a+b+c$
2) Perimeter of equilateral triangle = $3a$
3) Perimeter of square = $4a$
4) Perimeter of rectangle = $2(l+b)$
5) Perimeter of trapezium = $a+b+c+d$
6) Perimeter of rhombus = $4l$
7) Perimeter of regular hexagon = $6\times$(side)

8) Perimeter / circumference of circle $= 2\pi r$

9) Area of right angled triangle $= \frac{1}{2} \times$ base $\times$ height

10) Hero's formula : $A = \sqrt{S(S-a)(S-b)(S-c)}$

Where, $S = \frac{a+b+c}{2}$

11) Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{side})^2$

12) Area of square $= (\text{side})^2$

13) Area of rectangle $= (l \times b)$

14) Area of parallelogram $= h \times b$

15) Area of trapezium $= \frac{1}{2}$ (Sum of lengths of parallel sides) $\times$ height

16) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2$ ($\because d_1, d_2$ = diagona

17) Area of regular hexagon $= \frac{3\sqrt{3}}{2} \times (\text{side})^2$

18) Total surface area of sphere $= 4\pi r^2$

19) Surface area of hemisphere $= 3\pi r^2$

20) Surface area of cube $= 6 \times (\text{side})^2$

21) Total surface area of cuboid $= 2(lb + bh + lh)$

22) Surface area of hollow cylinder $= 2\pi rh$

23) Curved surface area of cone $= \pi r l$

24) Total surface area of cylinder $= 2\pi r(r+h)$

25) Volume of sphere $= \frac{4}{3}\pi r^3$

26) Volume of hemisphere = $\frac{2}{3}\pi r^3$

27) Volume of cube = $(side)^3$

28) Volume of cuboid = lbh

29) Volume of cylinder = $\pi r^2 h$

30) Volume of cone = $\frac{1}{3}\pi r^2 h$

11] Arithmetic Progression (A.P.)

1) If $a, a+d, a+2d, a+3d, \ldots$ are the terms in A.P. then,

(i) General term of an A.P. -

$$t_n = a + (n-1)d$$

Where, a is first term
& d is common difference

2) The sum of first 'n' terms of A.P -

$$S_n = \frac{n}{2}\left[2a \pm (n-1)d\right]$$

3) $S_n = \frac{n}{2}\left[1^{st} \text{ term} + \text{last term}\right]$

12] Geometric Progression (G.P.)

If $a, ar, ar^2, ar^3 \ldots$ are the terms in G.P. then,

(i) General term of an G.P. -

$t_n = ar^{n-1}$ where, a is first term & r is common ratio

(ii) Sum of 'n' turms of G.P. -

$S_n = a\left(\frac{1-r^n}{1-r}\right)$, where $r < 1$

$= a\left(\frac{r^n-1}{r-1}\right)$, where $r > 1$

$= na$, where $r = 1$

(iii) Sum of ∞ terms of G.P. -

$S_\infty = \frac{a}{1-r}$