✳ TUTORIAL NO. 02 ✳

TOPIC : Matrix - I

TITLE : Illustrate examples on normal and Echelon form of a matrix.

Question : Reduce the following martrice to normal form and hence find the rank of matrix.

1. $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$\Rightarrow$ Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

By $R_2 - 2R_1$, $R_3 - 3R_1$

$$A \approx \begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - C_1$

$$A \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix}$$

By $\frac{-1}{5} C_2$, $\frac{1}{2} C_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

By $R_3 - R_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

By $C_3 - C_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

By $-1 R_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \approx [I_3]$$

this is in the standard for $[I_3]$. Thus rank of given marix is 3.

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2. $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

$\Rightarrow$ Given $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

By R_{13}.

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$$A \approx \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix}$$

By $R_2 - 3R_1$, $R_3 - 2R_1$

$$A \approx \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - C_1$, $C_4 - 2C_1$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$$

By R_{23}, $-R_2$, $-R_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 6 & 2 & 4 \end{bmatrix}$$

By $R_3 - 6R_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & -28 & -56 \end{bmatrix}$$

By $C_3 - 5C_2$, $C_4 - 10C_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -28 & -56 \end{bmatrix}$$

By $\frac{-1}{28} R_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

By $C_4 - 2C_3$

$$A \approx \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \approx [I_3 \; 0]$$

this is standard form $[I_3]$
Thus rank of matrix is 3.

3. $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$\Rightarrow$ Given,

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 - R_1$

$$A \approx \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

By $C_2 - 2C_1$, $C_3 - 3C_1$, $C_4 - 2C_1$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

By $-R_2$, $R_3 - R_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_3 - C_2$, $C_4 - 3C_2$

$$A \approx \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \approx \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

this is standard form $[I_2]$
thus rank of given matrix is 2

4. $$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$\Rightarrow$ Given

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

By R_{12},

$$A \approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 - 6R_1$

$$A \approx \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

By $C_2 + C_1$, $C_3 + 2C_1$, $C_4 + 4C_1$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

By $R_2 - R_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

By $R_3 - 4R_2$, $R_4 - 9R_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

By $C_3 + 6C_2$, $C_4 + 3C_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$$

By $\frac{1}{33}C_3$, $\frac{1}{22}C_4$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

By $R_4 - 2R_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_4 - C_3$

$$A \approx \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

this is in standard for $[I_3]$. thus rank of matrix is 3.

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5. $A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$

$\Rightarrow$ Given.

$$A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

By R_{12}

$$A \approx \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 5 & 7 & 12 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

By $R_2 - 3R_1$, $R_3 - 3R_1$

$$A \approx \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - 2C_1$, $C_4 - 3C_1$, $C_5 - 5C_1$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $-R_2$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_3 - C_2$, $C_4 - 2C_2$, $C_5 - 3C_2$

$$A \approx \left[\begin{array}{cc|ccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

this is standard form $[I_2]$. thus rank of given matrix is 2.

6. $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -9 & 15 \\ 8 & -12 & 20 \end{bmatrix}$

$\Rightarrow$ Given that

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -9 & 15 \\ 8 & -12 & 20 \end{bmatrix}$$

By $R_2 - 3R_1$, $R_3 - 4R_1$

$$A \approx \begin{bmatrix} 2 & -3 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

By $C_2 + \frac{3}{2} C_1$, $C_3 - \frac{5}{2} C_1$.

$$A \approx \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence rank of the matrix A is 1.

7. $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$

$\Rightarrow$ Given that,

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

By $R_2 - R_1$, $R_3 - R_2$, $R_4 - R_3$

$$A \approx \begin{bmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 5 & 5 & 5 & 5 \end{bmatrix}$$

By $R_1 - 2R_2$, $R_3 - R_2$, $R_4 - 5R_2$

$$A \approx \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_{12}

$$A \approx \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - C_2$, $C_4 - C_3$.

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_3 - C_2$, $C_4 - C_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank of matrix A is 2.

(23) 8.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 & 3 \\ 1 & -1 & -2 & -4 & -6 \\ 3 & 1 & 3 & -2 & 5 \\ 6 & 3 & 0 & -7 & 2 \end{bmatrix}$$

$\Rightarrow$ By R_{12}

$$A \approx \begin{bmatrix} 1 & -1 & -2 & -4 & -6 \\ 2 & 3 & -1 & -1 & 3 \\ 3 & 1 & 3 & -2 & 5 \\ 6 & 3 & 0 & -7 & 2 \end{bmatrix}$$

By $C_2 + C_1$, $C_3 + 2C_1$, $C_4 + 4C_1$, $C_5 + 6C_1$.

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 5 & 3 & 7 & 15 \\ 3 & 4 & 9 & 10 & 23 \\ 6 & 9 & 12 & 17 & 38 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 - 6R_1$.

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 & 15 \\ 0 & 4 & 9 & 10 & 23 \\ 0 & 9 & 12 & 17 & 38 \end{bmatrix}$$

By $R_4 - (R_2 + R_3)$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 & 15 \\ 0 & 4 & 9 & 10 & 23 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_4 - (\frac{1}{3} C_3 + 2C_2)$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 & 15 \\ 0 & 4 & 3 & 0 & 23 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_5 - 3C_2$ $C_3 - 3C_5$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_2 - 5C_3$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By $C_2 - \frac{4}{3} C_5$, $\frac{1}{3} C_5$

$$A \approx \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By C_{23}, C_{35}

$$A \approx \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ Rank of matrix $A = 3$.

Question : Use elementary transformation to reduce the following matrix to echelon form.

1. $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$

$\Rightarrow$ Given matrix is,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

By $R_2 + R_1$, $R_3 - 2R_1$

$$A \approx \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 3 \\ 0 & -3 & -5 \end{bmatrix}$$

By $\frac{1}{2} R_2$

$$A \approx \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3/2 \\ 0 & -3 & -5 \end{bmatrix}$$

By $R_3 + 3R_2$

$$A \approx \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3/2 \\ 0 & 0 & -1/2 \end{bmatrix}$$

which is in echelon form and number of non zero rows are 3.

Hence $\rho(A) = 3$.

2. $A = \begin{bmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -3 \end{bmatrix}$

$\Rightarrow$ Given matrix is,

$$A = \begin{bmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -3 \end{bmatrix}.$$

By R_{14}

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 3 & 4 & 1 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 + R_1$, $R_4 - 3R_1$

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 6 & -1 & 12 \\ 0 & -3 & 8 & 1 \\ 0 & 7 & -5 & 10 \end{bmatrix}$$

By $\frac{1}{6} R_2$

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 1 & -1/6 & 2 \\ 0 & -3 & 8 & 1 \\ 0 & 7 & -5 & 10 \end{bmatrix}$$

By $R_3 + 3R_2,\ R_4 - 7R_2$

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 1 & -1/6 & 2 \\ 0 & 0 & 15/2 & 7 \\ 0 & 0 & -23/6 & -4 \end{bmatrix}$$

By $\frac{2}{15} R_3$

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 0 & -1/6 & 2 \\ 0 & 0 & 1 & 14/15 \\ 0 & 0 & -23/6 & -4 \end{bmatrix}$$

By $R_4 + \frac{23}{6} R_3$

$$A \approx \begin{bmatrix} 1 & -1 & 2 & -3 \\ 0 & 0 & -1/6 & 2 \\ 0 & 0 & 1 & 14/15 \\ 0 & 0 & 0 & -19/45 \end{bmatrix}$$

which is in echelon form and number of non zero rows are 4.

Hence $\rho(A) = 4$.

3.

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

$\Rightarrow$ Given matrix is.

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 2 \\ 2 & -1 & 2 & 5 \\ 5 & 6 & 3 & 2 \\ 1 & 3 & -1 & -3 \end{bmatrix}$$

By $R_2 - 3R_1,\ R_3 - 2R_1,\ R_4 - 5R_1,\ R_5 - R_1$

$$A \approx \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & -4 & -2 & 2 \\ 0 & -5 & 0 & 5 \\ 0 & -4 & -2 & 2 \\ 0 & 1 & -2 & -3 \end{bmatrix}$$

By R_{25}

$$A \approx \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & -5 & 0 & 5 \\ 0 & -4 & -2 & 2 \\ 0 & -4 & -2 & 2 \end{bmatrix}$$

By $R_3 + 5R_2,\ R_4 + 4R_2,\ R_5 + 4R_2$

$$A \approx \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -10 & -10 \\ 0 & 0 & -10 & -10 \\ 0 & 0 & -10 & -10 \end{bmatrix}$$

By $R_4 - R_2$, $R_5 - R_3$

$$A \approx \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -10 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which is in echelon form and number of non zero rows are 3

Hence $\rho(A) = 3$.

4. $A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow$ Given matrix is

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & -1 & 1 & -1 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 + R_1$, $R_4 - 2R_1$.

$$A \approx \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -2 & 1 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{bmatrix}$$

By R_{23}

$$A \approx \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & -3 & 1 & -2 \end{bmatrix}$$

By $R_3 + 2R_2$, $R_4 + 3R_2$

$$A \approx \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

By $R_4 - R_3$

$$A \approx \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

which is in echelon form, and number of non zero rows are 4.

$\therefore \rho(A) = 4$.

5. $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

$\Rightarrow$ Given matrix is

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

By $R_2 + 2R_1$, $R_3 - R_1$

$$A \approx \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

By $R_2 - 3R_4$, $R_3 + 2R_4$

$$A \approx \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

By R_{24}.

$$A \approx \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

$\therefore$ Rank of matrix $A = 2$.

6. $A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$

$\Rightarrow$ Given matrix is,

$$A = \begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$$

By $R_2 - R_1$, $R_3 - 2R_1$, $R_4 - 3R_1$

$$A \approx \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & -2 & -1 & 0 \\ 0 & -2 & -2 & 2 \end{bmatrix}$$

By $R_4 - 2R_2$, $R_3 - 2R_2$

$$A \approx \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in row echelon form. The number of non zero rows are 3.

$\therefore$ Rank of matrix $A = 3$.

7. $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 0 \end{bmatrix}$

$\Rightarrow$ Given matrix is

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

By $R_2 - 3R_1$, $R_3 - R_1$

$$A \approx \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

By R_{23}

$$A \approx \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in echelon form and number of non zero rows are 2

$\therefore$ Rank of matrix $A = 2$.

8. $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$ (2)

$\Rightarrow$ Given matrix is

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

By $R_2 - R_1$, $R_3 - R_1$

$$A \approx \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix}$$

By $R_3 - 3R_1$

$$A \approx \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is echelon form and number of non zero rows are 2

$\therefore$ Rank of matrix $A = 2$.

17. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ find two matrices P & Q such that PAQ is in normal form. Also find the rank of matrix A.

$\Rightarrow$ We first write, $A = I_3 A I_3$

i.e. $$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - R_1$, $R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $\frac{-1}{2} R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} = PAQ$$

$$\therefore P = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -2 & -1 & 1 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$\therefore$ The rank of matrix $A = 2$

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18. Find non singular matrices P & Q such that PAQ is in normal form, where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$. Also find rank of matrix A.

$\Rightarrow$ Let $A = I_3 A I_4$

$$\therefore \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$, $R_3 - 3R_1$.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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By $\frac{1}{12}R_3$.

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\2&-1&0\\-1/12&1/6&-1/12\end{bmatrix} A \begin{bmatrix}1&1&-5&-9\\0&1&-2&-5\\0&-1&3&5\\0&0&0&1\end{bmatrix}$$

By C_{34}.

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\2&-1&0\\-1/12&1/6&-1/12\end{bmatrix} A \begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$$

this is of the form $[I_3\ 0] = PAQ$,

where $P = \begin{bmatrix}1&0&0\\2&-1&0\\-1/12&1/6&-1/12\end{bmatrix}$ and $Q = \begin{bmatrix}1&1&-9&-5\\0&1&-5&-2\\0&-1&5&3\\0&0&1&0\end{bmatrix}$

19. If $A = \begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$ determine two non singular matrices P and Q such that $PAQ = I$.

$\Rightarrow$ Let $A = I_3 A I_3$

$$\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} A \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

By $R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-C_2$, R_{23}

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $R_3 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & -3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By $C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & -3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

This is of the form $[I_3] = PAQ$, where

$$P = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ -2 & 3 & -3 \end{bmatrix} \quad \text{and} \quad Q \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

20. Find non-singular matrices P & Q such that PAQ is in normal form where $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

$\Rightarrow$ Let $A = IAI$

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By R_{13}

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 3R_1$, $R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By $C_2 - C_1$, $C_3 - C_1$, $C_4 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By $-R_2,\ -R_3,\ R_{23}$,

$$\begin{bmatrix}1&0&0&0\\0&1&5&10\\0&0&2&4\end{bmatrix}=\begin{bmatrix}0&0&1\\-1&0&2\\0&-1&3\end{bmatrix}A\begin{bmatrix}1&-1&-1&-2\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

By R_3-6R_2

$$\begin{bmatrix}1&0&0&0\\0&1&5&10\\0&0&-28&-56\end{bmatrix}=\begin{bmatrix}0&0&1\\-1&0&2\\6&-1&-9\end{bmatrix}A\begin{bmatrix}1&-1&-1&-2\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

By $C_3-5C_2,\ C_4-10C_2$.

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-28&-56\end{bmatrix}=\begin{bmatrix}0&0&1\\-1&0&2\\-6&-1&-9\end{bmatrix}A\begin{bmatrix}1&-1&4&8\\0&1&-5&-10\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

By $\frac{-1}{28}R_3$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&2\end{bmatrix}=\begin{bmatrix}0&0&1\\-1&0&2\\-6/28&1/28&9/28\end{bmatrix}A\begin{bmatrix}1&-1&4&8\\0&1&-5&-10\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

By C_4-2C_3

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}=\begin{bmatrix}0&0&1\\-1&0&2\\-6/28&1/28&9/28\end{bmatrix}A\begin{bmatrix}1&-1&4&0\\0&1&-5&0\\0&0&1&-2\\0&0&0&1\end{bmatrix}$$

This is of the form $[I_3\ 0]=PAQ$

where. $P=\begin{bmatrix}0&0&1\\-1&0&2\\-6/28&1/28&9/28\end{bmatrix}$ and $Q=\begin{bmatrix}1&-1&4&0\\0&1&-5&0\\0&0&1&-2\\0&0&0&1\end{bmatrix}$